

# *The Motion of Electrons in Gases.*

By Prof. J. S. TOWNSEND, F.R.S., and H. T. TIZARD.

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1. The methods of investigating the motion of negative ions in gases at low pressure that have been explained in some previous papers may be extended to cases in which larger variations are made in the electric force and pressure. In order to find the kinetic energy of the motion of agitation of the ions, the velocity in the direction of an electric force, and the value of  $e/m$  for different forces and pressures, it is necessary to investigate experimentally two properties which are characteristic of the motion of electrons. These are the abnormal lateral diffusion of a stream of ions moving in a uniform electric field, and the deflection of the stream produced

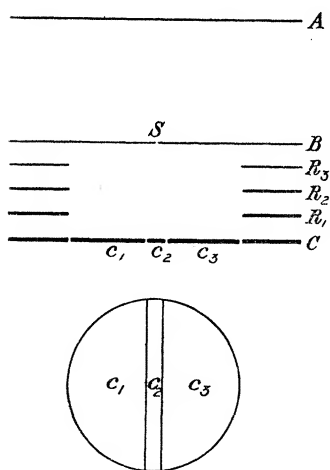


FIG. 1.

by a small transverse magnetic force. In the previous experiments the two phenomena were investigated separately and in each case with apparatus which gave satisfactory results when small electric forces were used and the pressures were limited to a certain range. In order to investigate the motion under larger forces an apparatus of more suitable dimensions was constructed, by means of which both the required sets of experiments may be made.

2. The negative ions were generated by the action of ultra-violet light on the plate A, fig. 1, and after traversing the distance from A to B some of the ions passed through a narrow slit S, 2 mm. wide and 15 mm. long, in the centre of the metal sheet B. The electric force was in the same direction on the two sides of B, so that the ions, after passing through the slit, continue their motion towards the plane electrodes C, which were parallel to the plane of B. The electrodes C were 4 cm. from B, and three flat rings, R<sub>1</sub>, R<sub>2</sub>, R<sub>3</sub>, 7 cm. internal diameter, were fixed at distances of 1, 2, and 3 cm. respectively from the plane of the electrode C. A separate connection for each ring and for the plates A and B was brought out through a large ebonite plug fitted in the brass cover of the apparatus, and was maintained at a potential proportional to the distance of the corresponding ring, or plate, from the electrodes C. The stream of ions

that came through the slit moved in a uniform electric field and was received by the three insulated electrodes  $c_1$ ,  $c_2$ , and  $c_3$ . These were portions of a disc 7 cm. in diameter, the central section  $c_2$  being 4.5 mm. wide and separated from the two equal side plates  $c_1$  and  $c_3$  by air gaps 0.5 mm. wide. The narrow gaps between the electrodes were parallel to the direction of the slit in B. In the calculations it will be supposed that the electrode  $c_2$  is 5 mm. wide, and that the side plates  $c_1$  and  $c_3$  come within 2.5 mm. of the central line.

When no magnetic force is acting the charges  $n_1$  and  $n_3$  received by the electrodes  $c_1$  and  $c_3$  were equal and the centre of the stream fell on the centre of  $c_2$ . If  $n_2$  be the charge received by  $c_2$  the ratio  $n_2/(n_1+n_2+n_3)$  depends on the electric force  $Z$  and the velocity of agitation of the ions.

In the previous experiments the factor  $k$  by which the energy of agitation of the ions exceeded that of the surrounding molecules was deduced from observations of the lateral diffusion of a stream of circular section after traversing a distance of 7 cm. under the action of the electric force. The stream was received on a small circular electrode in the centre of a large metal ring, so that the proportion of the charge received by the small disc diminishes rapidly as the motion of agitation increases. The apparatus was therefore suitable for determining the smaller values of the quantity  $k$ .

The apparatus described above was better suited for the determination of large increases in the velocity of agitation and the values of  $k$  may easily be found from the ratio  $n_2/(n_1+n_2+n_3)$ , as is seen from the following investigation.

3. Let the origin of co-ordinates be taken in the centre of the slit, the axis of  $z$  being normal to the plate B and the axis of  $y$  parallel to the length of the slit. Since the central electrode  $c_2$  is much longer than the slit the diffusion in the direction  $y$  does not affect the number of ions that are received on the electrode  $c_2$ , and it will only be necessary to find the motion in the directions  $x$  and  $z$ .

Considering the ions that pass at a uniform rate through a narrow section in the centre of the slit parallel to the axis of  $y$ , the distribution in the electric field when the motion becomes steady is given by the equation\*

$$\nabla^2 n = \frac{Ne}{\Pi} \cdot \frac{Z}{k} \cdot \frac{dn}{dz}.$$

When the electric force  $Z$  is measured in volts per centimetre, and the values of the constants  $Ne$  and  $\Pi$  are substituted, the equation becomes

$$\nabla^2 n = 41 \frac{Z}{k} \cdot \frac{dn}{dz}.$$

\* J. S. Townsend, 'Roy. Soc. Proc.,' 1908, A, vol. 81, p. 469.

At the electrodes C ( $z = 4$ ) the number of ions received on an area between the parallel lines  $x = 0$  and  $x = a$  is proportional to  $\int_0^a \int_0^b n \, dy \, dx$ .

As the ions move in a very wide space they do not extend to the rings R forming the boundary, so that when integrating along the axis of  $y$  from one end of the field to the other, the values of  $n$  and  $dn/dy$  are zero at the boundary. The above equation on integration with respect to  $y$  becomes

$$\frac{d^2q}{dx^2} + \frac{d^2q}{dz^2} - \frac{41Z}{k} \cdot \frac{dq}{dz} = - \left[ \frac{dn}{dy} \right] = 0,$$

where  $q = \int n \, dy$ , the integration being taken between two points on the boundary.

The series of sines that represents the solution of the equation converges very slowly owing to the nature of boundary conditions, so that it is necessary to express the solution in a more practical form.

Since the principal motion is in the direction of the electric force  $Z$ , the ions move in a narrow stream when  $Z/k$  is sufficiently large, and the quantities  $dq/dz$  and  $d^2q/dz^2$  are small compared with  $d^2q/dx^2$ . For a first approximation the value of  $q$  may therefore be obtained from the equation

$$\frac{d^2q}{dx^2} - \frac{41Z}{k} \cdot \frac{dq}{dz} = 0.$$

With the values of  $Z/k$  that occur in the experiments the error introduced by neglecting the term  $d^2q/dz^2$  is probably smaller than the experimental error. It thus appears that the distribution of the quantity  $q$  as expressed in terms of  $x$  and  $z$  is the same as the distribution of temperature  $\phi$ , in terms of  $x$  and  $t$ , in an infinite solid initially at zero temperature throughout, except at the plane  $x = 0$  where the temperature has a constant value  $\phi_0$  when  $t = 0$ . The temperature is obtained from the equation

$$\frac{d^2\phi}{dx^2} = \frac{1}{K} \cdot \frac{d\phi}{dt},$$

and the solution given by Fourier is  $\phi = At^{-\frac{1}{2}} e^{-x^2/4Kt}$ .

In the problem of the distribution of ions in the space between the plates B and C the surface conditions are  $q = 0$  when  $z = 0$  for all values of  $x$  except  $x = 0$ , so that  $q$  in terms of  $x$  and  $z$  is given by the equation

$$q = Az^{-\frac{1}{2}} e^{-\frac{41Z}{4kz} x^2}.$$

The distance  $z$  of the electrodes  $c$  from the origin was 4 cm. and the central electrode was 5 mm. wide, so that the ratio  $R$ , of the charge  $n_2$  received by the central electrode to the total charge  $n_1 + n_2 + n_3$ , is

$$R = \frac{n_2}{n_1 + n_2 + n_3} = \frac{\int_0^{0.25} e^{-2.56 Z_1 x^2} dx}{\int_0^{\infty} e^{-2.56 Z_1 x^2} dx} = \frac{\int_0^{0.4 \sqrt{Z_1}} e^{-y^2} dy}{\int_0^{\infty} e^{-y^2} dy},$$

where  $Z_1 = Z/k$ .

When various values are given to the quantity  $Z_1$  the corresponding ratios  $R$  may be obtained from the tables of the values of the integrals.

It is necessary to take into consideration the width of the slit, since with the larger values of  $Z/k$  the proportion of the ions coming through a section of the slit near the edge that arrive on the central electrode is somewhat less than the proportion of those that come through at the centre. The exact proportion for any section of the slit is easily calculated and the ratio  $R = n_2/(n_1 + n_2 + n_3)$  when the ions come through all sections of the slit equally may be found in terms of  $Z$ .

The curve, fig. 2, gives the values of  $R$  in terms of  $Z_1 = Z/k$ .

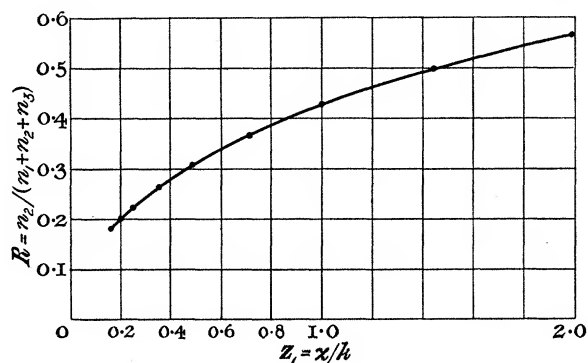


FIG. 2.

4. When the ratio  $R$  is determined experimentally with an electric force  $Z$  acting between B and C,  $Z_1$  may be found from the curve, fig. 2, and the corresponding value of  $k = Z/Z_1$  may be determined.

The curves, fig. 3, give the values of  $R = n_2/(n_1 + n_2 + n_3)$  found for air at various pressures. The ratio  $R$  corresponding to a given force and pressure depends on the dryness of the gas. When the pressure is reduced to the required value,  $R$  may be observed at intervals while the small quantity of moisture in the apparatus is removed slowly by the phosphorus pentoxide. As the air dries the value of  $R$  diminishes and after some weeks a constant minimum value is attained. The continuous curves, fig. 3, represent the minimum values obtained for dry air with different forces and pressures.

Several experiments were made with air which had not been dried very completely. The values of  $R$  are then practically the same as those obtained

with dry air when the larger forces are acting. With the smaller forces the moisture reduces the divergence of the stream and comparatively large values of  $R$  are obtained. For a certain range of small forces the quantity  $k-1$

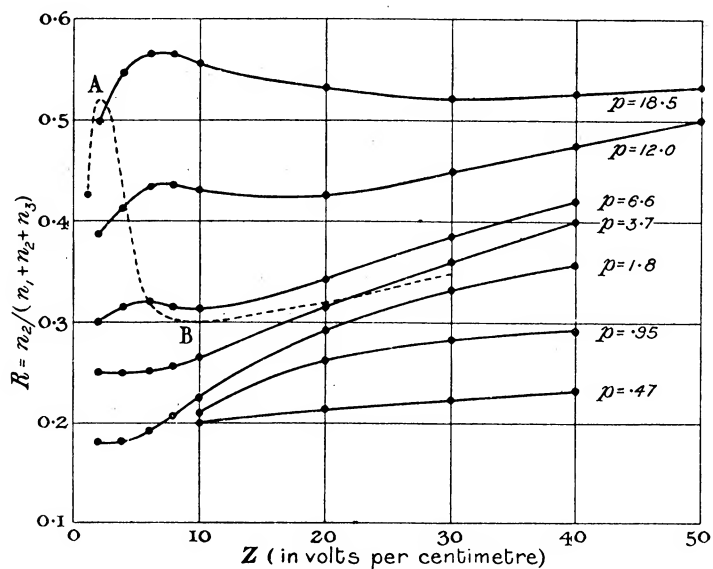


FIG. 3.

becomes very small, and the curve giving  $R$  in terms of  $Z$  coincides with the theoretical curve, fig. 2. In these cases  $R$  is independent of the pressure and the amount of water vapour present, provided the amount exceeds a certain small value.

The dotted curve, fig. 3, illustrates the behaviour of air containing a small quantity of moisture, the pressure being 2.7 mm. With a force of 1 volt per centimetre ( $Z = 1$ ),  $R = 0.425$ , which is the value of  $R$  in the theoretical curve at the point  $Z/k = 1$ . The two curves continue to coincide for some distance as  $Z$  increases, but when  $Z$  becomes 2 volts per centimetre,  $R$ , as determined experimentally, is 0.52, the theoretical curve giving 0.565 when  $Z/k = 2$ . The value of  $Z/k$  corresponding to the ratio 0.52 is 1.6, so that  $k = 1.25$  when  $Z = 2$ . The value of  $R$  reaches a maximum at A ( $Z = 2$ ), and an increase in the force is accompanied by a remarkable increase in the divergence of the stream. The ratio  $R$  attains a minimum value 0.3 at the point B where  $Z = 8$ , the value of  $k$  being  $8/0.46 = 17.4$ . Further increases in the force cause the divergence to diminish, and gradually the dotted curve takes a place between the two curves for dry air at 3.7 and 1.8 mm. pressure. For values of  $Z$  exceeding 30 the values of  $R$  are practically the same as for dry air at 2.7 mm. pressure.

This shows that the electrons adhere to the water molecules and the velocity of agitation of the charged particles is normal, when the force is small. For large forces the effect of the water vapour disappears, and the electrons move freely with a kinetic energy of agitation exceeding that of the surrounding molecules by the factor  $k$ . In the particular example illustrated by the dotted curve the effect of the water vapour disappears rapidly between the points A and B and when the force exceeds 30 volts per centimetre the electrons move as in dry air.

When the amount of water vapour increases, the ratios R and the forces Z, corresponding to the maximum and minimum points A and B, increase.

For different pressures  $p$ , when the amount of moisture is approximately proportional to the total pressure, the forces corresponding to the points A and B are also approximately proportional to the pressure. This is interesting, from a theoretical point of view, since these properties of the electrons depend on the velocity acquired between collisions with molecules, so that any particular effect should be obtained with the same value of  $Z/p$  if the proportion of the different gases that are present is not altered when the total pressure is altered.

5. The values of  $k$  for dry air were obtained from the curves, fig. 3, and it was found that  $k$  depends on the ratio  $Z/p$ . This is shown by the following examples of the numbers obtained from experiments at different pressures when  $Z/p$  is approximately constant:—

| $p$ . | Z. | $Z/p$ . | $k$ . | W.                 |
|-------|----|---------|-------|--------------------|
| 18.5  | 40 | 2.16    | 24.0  | $1.75 \times 10^6$ |
| 12.0  | 30 | 2.5     | 26.0  | $1.97 \times 10^6$ |
| 1.8   | 4  | 2.2     | 24.0  | $1.9 \times 10^6$  |
| 3.7   | 40 | 10.8    | 46.0  | $5.5 \times 10^6$  |
| 1.8   | 20 | 11.1    | 46.5  | $5.6 \times 10^6$  |
| 0.95  | 10 | 10.5    | 45.5  | $5.4 \times 10^6$  |

The agreement between the different determinations for values of  $Z/p$  between 0.2 and 0.5 was not as accurate as with the higher values. This is probably due to the fact that small quantities of impurities have more effect when the lower forces are acting.

The results of the experiments on the lateral diffusion may be represented by a single curve giving  $k$  as a function of  $Z/p$ . The smaller values of  $k$  from  $Z/p = 0.2$  to  $Z/p = 20$  are given in the curve, fig. 4, and the larger values from  $Z/p = 20$  to  $Z/p = 200$  are given in the curve, fig. 5.

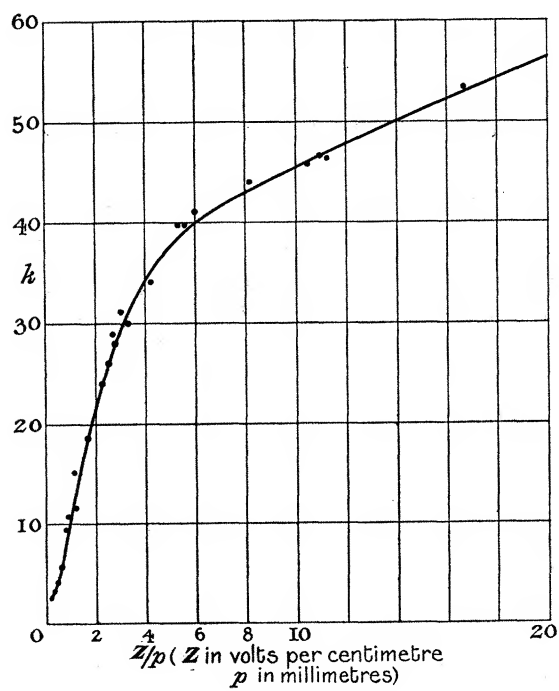


FIG. 4.

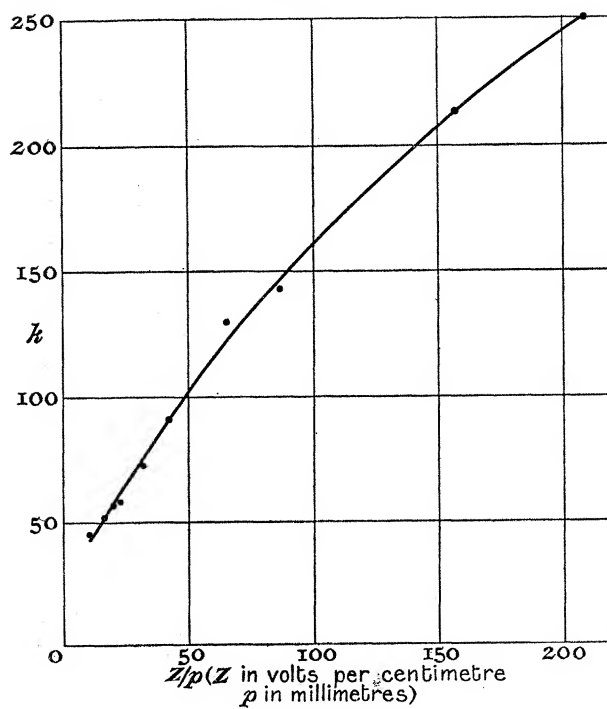


FIG. 5.

The results thus obtained are in good agreement with previous determinations obtained with negative ions generated by Röntgen rays, and the values obtained by Haselfoot for ions generated by ultra-violet light.

The conclusions to be drawn from these experiments depend on certain formulæ that have been obtained for cases in which the velocity of agitation of the ions is large compared with the velocity under an electric force. It will be seen from these investigations that when  $Z/p$  exceeds 0.2 the ions in dry air are in the electronic state, so that the value of  $e/m$  may be taken as  $5.3 \times 10^{17}$ . The value of  $e/m$  for a molecule of air being  $10^{13}$ , the masses are in the proportion  $5.3 \times 10^4 : 1$ , so that the velocity of agitation of an electron in thermal equilibrium with air is 230 times the velocity of agitation of a molecule of air, or  $10^7$  cm. per second. The actual velocities of agitation when electric forces are acting exceed this value by the factor  $\sqrt{k}$ . In order that the ordinary formulæ derived from the kinetic theory of gases should apply to the motion of the electrons it is necessary that the velocity  $W$  in the direction of the electric force  $Z$  should be considerably less than the quantity  $\sqrt{k} \times 10^7$ .

6. The velocities  $W$  were determined with the same apparatus for the same forces  $Z$  and pressures  $p$ .

A transverse magnetic force  $G$  was produced by a current in a pair of large circular coils outside the apparatus, the direction of the force being parallel to the slits. The stream was then deflected so that the centre no longer fell on the centre of the electrode  $c_2$  and the charge acquired by the electrode  $c_1$  increased with the force  $G$ . In these experiments the electrodes  $c_2$  and  $c_3$  were joined and the current in the coils was adjusted until the charge  $n_2 + n_3$  received by the electrodes  $c_2$  and  $c_3$  was equal to the charge  $n_1$  acquired by the electrode  $c_1$ . The centre of the stream was thus deflected through 2.5 mm. (half the width of the central plate) while the electrons travelled 4 cm. in the direction of the electric force.

The magnetic forces  $G$  required to deflect the centre of the stream through the angle  $\tan^{-1} 0.0625$  when the ions are moving under the electric force  $X$  are given by the curves (fig. 6). The different curves correspond to the different pressures of the air, which varied from 0.47 to 18.5 mm.

The velocity  $W$  under the electric force is given by the equation

$$GW/Z = \tan \theta = 0.0625,$$

which expresses the condition that the direction of motion of the centre of the stream is along the resultant of the forces  $Z \times e$  and  $G \times We$ . The latter force being small, the velocities  $W$  may be assumed to be the same as those in the experiments on the lateral diffusion when the forces  $Z$  were acting.



The values of  $W$  obtained from the formula  $W = (Z/G) \times 0.0625$  were found to depend on the ratio  $Z/p$ . This may be seen from the numbers in the last column of the table given in the preceding section. The values of  $W$  are expressed in centimetres per second, for the different forces and pressures.

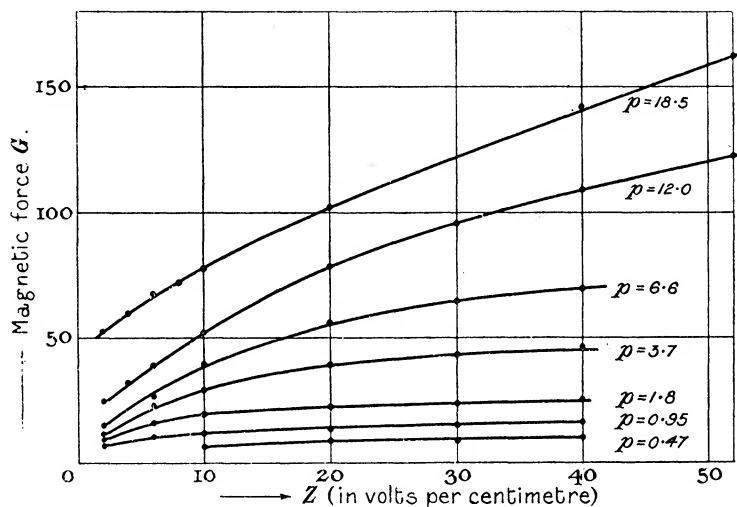


FIG. 6.

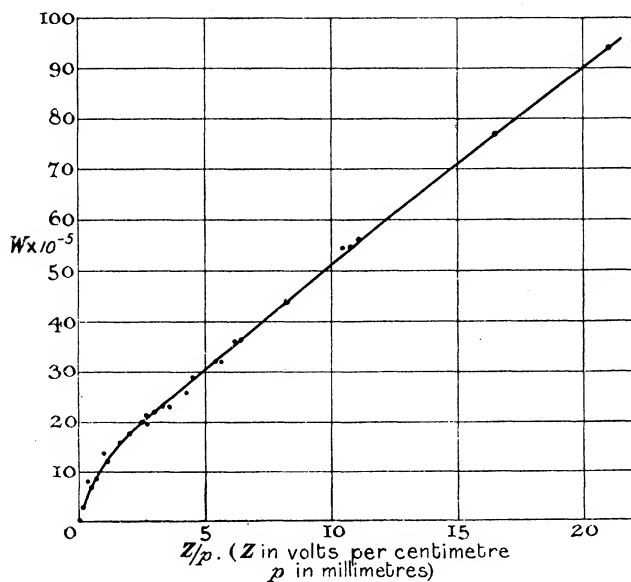


FIG. 7.

It follows that  $W$  is a function of  $Z/p$  and the velocities may be expressed by means of a single curve. The portion of the curve corresponding to values

of  $Z/p$  from 0.2 to 20 is given in fig. 7, and the remainder from  $Z/p = 20$  to  $Z/p = 200$  in fig. 8.

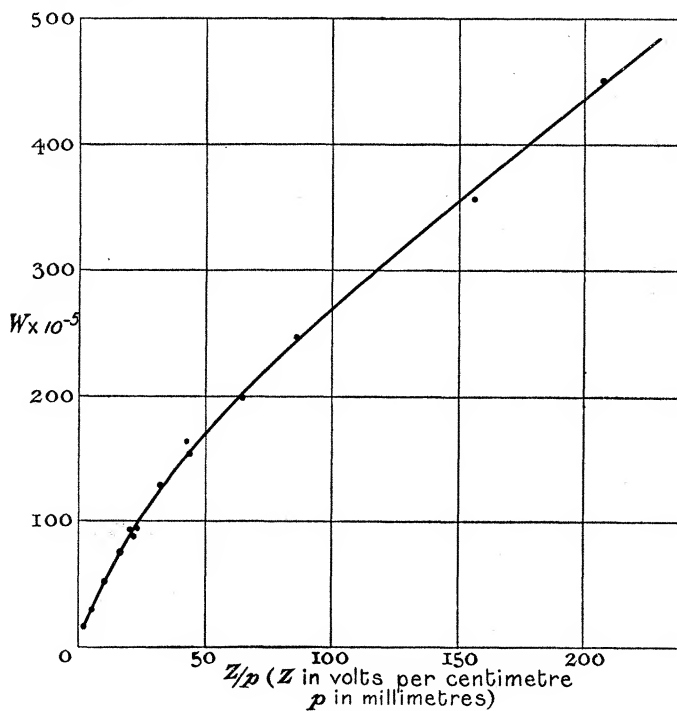


FIG. 8.

7. The following table gives a comparison between the velocity of agitation  $u$  and the velocity  $W$  in the direction of the electric force, in terms of  $Z/p$  :—

| $Z/p$ .....                     | 0.2 | 0.5 | 1    | 2    | 5   | 10  | 20  | 50   | 100  | 150  | 200  |
|---------------------------------|-----|-----|------|------|-----|-----|-----|------|------|------|------|
| $W \times 10^{-6}$ .....        | 0.5 | 0.9 | 1.25 | 1.75 | 3.0 | 5.2 | 9.0 | 17.3 | 27   | 35   | 44   |
| $u \times 10^{-6}$ .....        | 16  | 24  | 34   | 47   | 62  | 67  | 75  | 101  | 127  | 145  | 156  |
| $W \times u \times 10^{-12}$ .. | 8.0 | 21  | 42   | 82   | 186 | 350 | 675 | 1740 | 3430 | 5100 | 6900 |
| $e/m \times 10^{-17}$ .....     | 5.1 | 5.8 | 5.8  | 5.4  | 4.4 | 3.9 | 3.6 | 3.9  | 3.8  | 3.6  | 3.8  |

The velocity  $W$  of a charged particle moving under an electric force is given approximately by the formula  $W = (Ze/m)(l/u)$ , when the mass  $m$  of the particle is small compared with that of a molecule of the gas, and the velocity of agitation  $u$  is large compared with  $W$ ,  $l$  being the mean free path of the particle. When the pressure is constant  $l$  is also constant, and the velocity  $W$  is proportional to  $Z$  when  $m$  and  $u$  are independent of  $Z$ . In these cases  $m$  is constant, being the mass of an electron, but  $u$  increases with  $Z$  so that  $W$  does not increase in proportion to the force. The results of the experiments are thus in accordance with the theory, since  $W$  increases less rapidly than  $Z$  when  $p$  is constant; in fact, the

product  $W \times u$  is approximately proportional to  $Z$ . The formula diminishes in accuracy as  $Z$  increases, since the velocity  $W$  approaches  $u$ .

In the above experiments a certain number of the ions that reach the electrodes  $c$  are generated by the collisions of the electrons with molecules of the gas. This does not affect the ratios of the charges acquired by the electrodes  $c_1, c_2, c_3$ , since the number of ions generated by collisions at each point of the gas is proportional to  $n$ , when the forces are not so large that the positive ions also generate others by collisions. The only effect of ionisation by collision in these experiments is to increase the charges  $n_1, n_2$ , and  $n_3$ , in the same proportion. With the larger pressures when  $Z/p$  is less than 20 the number generated by collisions is very small, but when the pressure is 0.47 mm. 60 per cent. of the ions are generated between the slit in B and the electrodes  $c$  when  $Z = 40$  volts per centimetre.

8. The values of  $e/m$  for the charged particle may be deduced from the expressions for the velocity  $W$ , or the deflection  $\theta$ , in terms of molecular quantities. The formulæ  $W = (Ze/m)T$ , and  $\tan \theta = (He/m)T$ , obtained under simplified conditions, only give approximate results, but a more complete investigation has been made by Langevin\* of the rate of diffusion, and the velocity under an electric force for the case in which the collisions between the charged particle and the molecules are of the type that occurs between elastic spheres. The expression for the velocity  $W$  under these conditions is

$$W = \frac{3Ze}{8\sigma^2N} \sqrt{\frac{h(m+m')}{\pi mm'}},$$

where  $m$  is the mass of the charged particle,  $m'$  that of a molecule of the gas,  $\sigma$  the sum of the radii of the particle and a molecule,  $N$  the number of molecules of the gas per cubic centimetre, and  $h$  three-fourths of the reciprocal of the kinetic energy of agitation of the particle or a molecule of the gas.

Since the mass of the electron is small compared with a molecule the ratio  $(m+m')/mm'$  reduces to  $1/m$ , and  $1/\pi\sigma^2N$  is the mean free path  $l$  of the electron. The velocity  $W$  of an electron thus becomes

$$W = (Ze/m)(l/u) 0.815,$$

$u$  being the velocity of agitation of the electron. The velocity  $W$  is therefore independent of the mass of the molecules and is proportional to  $l/u = T$ , the mean interval between collisions.

Langevin's formula is deduced on the hypothesis that the mean energy of agitation of the particle  $m$  is the same as that of a molecule of the gas. When the masses are of the same order of magnitude the interval between

\* P. Langevin, 'Ann. de Chim. et de Phys.,' 1905, (8), vol. 5, p. 245.

collisions is a function of the velocities of both particles, but when the particle  $m$  is very small compared with  $m'$  the time  $T$  is determined by the velocity of agitation of the particle  $m$ . Hence in dealing with electrons the velocity  $u$  must be taken as the actual velocity of agitation, which exceeds that corresponding to a particle in thermal equilibrium with the molecules of the gas by the factor  $\sqrt{k}$ .

The value of  $e/m$  for the electrified particle may be obtained from the above equation by eliminating the quantity  $u$ . Thus

$$W^2 = (Z^2 e/m) (Ne/mNu^2) l^2 \times [0.815]^2,$$

where  $Ne = 1.23 \times 10^{10}$  and  $mNu^2 = 3k \times 10^6$ . At a millimetre pressure the mean free path of a molecule of air may be taken as  $7.5 \times 10^{-6} \times 760$ , so that the mean free path of an electron is  $3.2 \times 10^{-2}$  cm. Hence  $lp = 3.2 \times 10^{-2}$  and  $e/m$  is given by the formula

$$e/m = (Wp/Z)^2 (k/2.8).$$

The values of  $e/m$  thus obtained from the determinations of  $W$  and  $k$  corresponding to the different values of  $Z/p$  are given in the above table (p. 345).

The numbers obtained for  $e/m \times 10^{-17}$  do not differ very much from the value 5.3 found by the more accurate methods under conditions in which the effects of collisions between electrons and molecules may be neglected.

The differences between the values found for  $e/m$  for different values of  $Z/p$  are not entirely due to experimental errors, although the probable error in the final results is greater than that in the direct measurements, since the square of the velocity  $W$  is involved in the formula for  $e/m$ . The nature of the collisions between electrons and molecules may not resemble the collisions between elastic spheres to the same extent for different values of the velocity  $u$ .\*

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\* Mr. F. B. Pidduck (p. 296, *supra*) has made a theoretical investigation of the motion of ions in gases; he shows that when the negative ions are in the electronic state their velocity of agitation may become abnormally large, and the above values of  $u$  and  $k$  may be explained on the hypothesis that the collisions between electrons and molecules are such as would occur between imperfectly elastic spheres.